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Origins of Hierarchical Logical Reasoning

Abhishek M. Dedhe,[a,b] Hayley Clatterbuck,[c] Steven T. Piantadosi,[d]
Jessica F. Cantlon[a,b]

[a]*Department of Psychology, Carnegie Mellon University*
[b]*Center for the Neural Basis of Cognition, Carnegie Mellon University*
[c]*Department of Philosophy, University of Wisconsin*
[d]*Department of Psychology, University of California*

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Abstract

Hierarchical cognitive mechanisms underlie sophisticated behaviors, including language, music, mathematics, tool-use, and theory of mind. The origins of hierarchical logical reasoning have long been, and continue to be, an important puzzle for cognitive science. Prior approaches to hierarchical logical reasoning have often failed to distinguish between observable hierarchical behavior and unobservable hierarchical cognitive mechanisms. Furthermore, past research has been largely methodologically restricted to passive recognition tasks as compared to active generation tasks that are stronger tests of hierarchical rules. We argue that it is necessary to implement learning studies in humans, non-human species, and machines that are analyzed with formal models comparing the contribution of different cognitive mechanisms implicated in the generation of hierarchical behavior. These studies are critical to advance theories in the domains of recursion, rule-learning, symbolic reasoning, and the potentially uniquely human cognitive origins of hierarchical logical reasoning.

Keywords: Logic; Rule-learning; Pattern recognition; Hierarchical reasoning; Bayesian modeling

1. Introduction

Hierarchical logical reasoning is central to the organization and implementation of sophisticated behaviors (Asano, Boeckx, & Seifert, 2021; Chomsky, 1956; Friederici, 2020;

Correspondence should be sent to Abhishek M. Dedhe, Department of Psychology, Carnegie Mellon University, 4909 Frew St, Pittsburgh, PA 15213, USA. E-mail: adedhe@andrew.cmu.edu

Greenfield, 1991; Greenfield & Schneider, 1977; Palmer, 1977; Thibault et al., 2021), including language, mathematics, music, tool-use, and theory of mind (Corballis, 2014; Greenfield, 1991; Hauser, Chomsky, & Fitch, 2002; Hofstadter, 1979; Laland & Seed, 2021; Maclean, 2016; Pinker & Jackendoff, 2005; Tomasello & Rakoczy, 2003). Hierarchical structures are composed of lower-level units combined to form higher-level ones (Greenfield, 1991). Constituent lower-level units are building blocks for higher-level units and can be flexibly combined and moved around (Fodor, 2001; Westphal-Fitch, Huber, Gomez, & Fitch, 2012). Patricia Greenfield (1991) hypothesized that widespread hierarchical behavior arising from hierarchical cognitive mechanisms is central to domain-general abstract cognition. Here, we identify two approaches that are essential for testing this hypothesis:

1. Comparisons of hierarchical versus non-hierarchical models are critical tests of the domain-generality of hierarchical cognitive mechanisms.
2. Generation (as opposed to recognition) of hierarchical structures is the key behavior for identifying hierarchical cognitive mechanisms.

2. Behavior and cognitive mechanisms

Hierarchical behavior, that is, the perception and production of hierarchical structures, differs from hierarchical cognitive mechanisms and mental representations. The fact that hierarchical behavior is seen across multiple domains is uncontroversial (Corballis, 2014; Fischmeister, Martins, Beisteiner, & Fitch, 2017; Fitch, 2014; Hauser & Watumull, 2017; Truswell, 2017; Vyshedskiy, 2019); the claim that hierarchical cognitive mechanisms underlie this behavior is not (Frank, Bod, & Christiansen, 2012; Lobina, 2014). Indeed, hierarchical behavior may arise from non-hierarchical cognitive mechanisms like statistical learning (Camp, 2009; Rey, Perruchet, & Fagot, 2012; Santolin & Saffran, 2018) or ordinal reasoning (D'amato & Colombo, 1990; McGonigle & Chalmers, 1977; Orlov, Yakovlev, Hochstein, & Zohary, 2000; Terrace & McGonigle, 1994). Hierarchical structures abound in nature; for example, primate societies tend to be hierarchically organized (Franz, McLean, Tung, Altmann, & Alberts, 2015). Non-human primates, such as baboons, can perceive social dominance hierarchies (Cheney & Seyfarth, 2008). However, this does not imply that hierarchical cognitive mechanisms are involved; other processes like linear transitive inference may lead to the emergence of dominance hierarchies (Camp, 2009; Franz et al., 2015; Paz-y-Miño, G., A., Kamil, & Balda, 2004). Hierarchical structures can thus exist in the absence of hierarchical processes. An open puzzle is whether complex behaviors require specialized hierarchical mechanisms (Corballis, 2007, 2014; Culbertson & Adger, 2014; Dehaene, Meyniel, Wacongne, Wang, & Pallier, 2015; Fitch, 2014; Greenfield, 1991; Hauser et al., 2002). This question has been mainly asked in the domains of language and music but is applicable to a wide range of hierarchical behavior, including mathematics, tool-use, visual pattern perception, goal-directed actions, reasoning and decision-making, and complex social cognition (Dehaene, Al Roumi, Lakretz, Planton, & Sablé-Meyer, 2022; Hauser et al., 2002).

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For decades, language learning studies (Fitch & Hauser, 2004; Miller, 1967; Reber, 1967) have dominated the hierarchical rule-learning literature. However, as McCoy, Culbertson, Smolensky, and Legendre (2021) describe in a recent review of artificial grammar learning paradigms, previous studies could not rule out that participants were using non-hierarchical strategies like attending to word order, bigrams, counting, or cognitive mechanisms like associative chaining or ordinal reasoning (De Vries, Monaghan, Knecht, & Zwitserlood, 2008; Ferrigno, Cheyette, Piantadosi, & Cantlon, 2020; Rey et al., 2012). Other researchers have highlighted how aspects of language comprehension are often assumed to require explicit hierarchical encoding but can be comprehended without hierarchical cognitive mechanisms or may arise from memory constraints (Cornish, Dale, Kirby, & Christiansen, 2017; Frank & Bod, 2011; Frank & Yang, 2018). A recent paper about hierarchical structure learning in infants by Shi, Emond, and Badri (2020) describes how most studies are not designed to contrast linear (i.e., non-hierarchical) versus hierarchical alternatives. In recent years, it has become clear that intelligent agents have multiple cognitive mechanisms at their disposal for solving even simple tasks. To identify what mechanisms underlie hierarchical behavior, we need a priori, ideally formal and mathematical, hypotheses about the expected behavior driven by different alternative cognitive mechanisms, viz., *p*(*behavior* | *cognitive mechanism*). For example, two recent papers formalized competing models of associative, ordinal, and hierarchical processes to test the mechanisms underlying human and monkey behavioral performance in a hierarchical reasoning task (Ferrigno et al., 2020; Lakretz & Dehaene, 2021). Another study showed evidence that multiple mechanisms can simultaneously influence hierarchical behavior (Dedhe et al., 2022a). Formulating these predictions in Bayesian terms is helpful (Fitch, 2014) because researchers can then use formal models to calculate *p*(*cognitive mechanism* | *behavior*)—the inferred probability that a given cognitive mechanism was involved in driving hierarchical behavior (Dedhe et al., 2022b). This approach allows for a more precise assessment of whether different cognitive mechanisms, both hierarchical and linear, interact with each other during a task. In summary, comparisons of hierarchical versus non-hierarchical models are critical tests of the domain-generality of hierarchical cognitive mechanisms.

3. Recognition and generation

Studies of natural behaviors suggest a shift in the mechanisms underlying hierarchical behavior over time. Every-day hierarchical behaviors occur in the domains of action and tool-use during routines and constructing complex objects. Children progress through a series of developmental stages from linear to hierarchical as they learn these everyday hierarchical routines (Beagles-Roos & Greenfield, 1979; Goodson & Greenfield, 1975; Greenfield, 1991; Greenfield & Childs, 1977; Greenfield & Schneider, 1977). Domains like language (Matthei, 1982) and theory of mind (Tomasello, 2018) also exhibit developmental progressions from linear to hierarchical rule use during early childhood. A major limitation of prior research about the development of hierarchical behavior is its focus on the recognition (i.e., comprehension) of hierarchical structures as opposed to their generation (i.e., production).

Recognition tasks typically involve forcing participants to choose between two alternatives, and the underlying mechanisms are often ambiguous (McCoy et al., 2021). In contrast, generation tasks involve the active step-by-step production of hierarchical structures that need to be selected from a large number of non-hierarchical alternatives. The robustness of hierarchical rule use is less apparent in tasks that do not require generation. Such recognition tasks might, therefore, tap into different cognitive mechanisms than tasks that do require generation. A handful of recent studies used generation tasks to test whether hierarchical cognitive mechanisms were implicated in behavior (Ferrigno et al., 2020; Jiang et al., 2018; Lake & Piantadosi, 2020; Malassis, Dehaene, & Fagot, 2020; Rey et al., 2012). These studies of hierarchical rule generation test not only whether an individual detects hierarchical patterns, but also whether they can use hierarchical logical reasoning to produce those structures, thus providing a stronger test of learning, rule-use, and generalization. In summary, generation (as opposed to recognition) of hierarchical structures is the key behavior for identifying hierarchical cognitive mechanisms.

4. Conclusion

The hypothesis that diverse hierarchical behaviors, including language, music, mathematics, tool-use, visual pattern perception, goal-directed actions, reasoning & decision-making, and complex social cognition arise from domain-general hierarchical cognitive mechanisms, has been a mainstay of cognitive science over the past few decades. However, prior approaches to hierarchical logical reasoning have often failed to distinguish between observable hierarchical behavior and unobservable hierarchical cognitive mechanisms. Furthermore, past research has been largely methodologically restricted to passive recognition tasks as compared to active generation tasks that are stronger tests of the use of hierarchical rules. These two limitations need to be addressed to test the domain-generality of hierarchical cognitive mechanisms. We therefore argue that it is necessary to implement learning studies in humans, non-human species, and machines that are analyzed with formal models comparing the contribution of different cognitive mechanisms implicated in the generation of hierarchical behavior.

Hierarchical generation tasks are often difficult for non-humans to learn. Non-human primates generate hierarchical structures in some tasks, but only after extensive training (Ferrigno et al., 2020; Rey et al., 2012). In the wild, non-human vocal sequences typically lack hierarchical structure (Girard-Buttoz et al., 2022; Townsend, Engesser, Stoll, Zuberbühler, & Bickel, 2018), while the hierarchical sequences that are generated by songbirds and whales may be devoid of meaning and rich semantic content (Sainburg, Theilman, Thielk, & Gentner, 2019; Suzuki, Buck, & Tyack, 2006). Machines also struggle to learn hierarchical structures (Elman, 1990; Kirov & Frank, 2012; Larketz et al., 2021; though see Murty et al., 2022; Yang & Piantadosi, 2022). Furthermore, machines better approximate human-like hierarchical rules when endowed with inductive biases toward inferring hierarchies like nested-tree structures (Coopmans, De Hoop, Kaushik, Hagoort, & Martin, 2022; Lake & Piantadosi, 2020). In summary, differences between species and between natural versus artificial

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intelligences suggest that humans are "dendrophiliacs," showing a widespread proclivity toward hierarchical psychological processes (Dehaene et al., 2022; Fitch, 2014). Further research is needed to shed light onto the potentially uniquely human origins of domain-general hierarchical logical reasoning, an important outstanding puzzle for cognitive science.

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