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The Cultural Origins of Symbolic Number

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It is popular in psychology to hypothesize that representations of exact number are innately determined—in particular, that biology has endowed humans with a system for manipulating quantities which forms the primary representational substrate for our numerical and mathematical concepts. While this perspective has been important for advancing empirical work in animal and child cognition, here we examine six natural predictions of strong numerical nativism from a multidisciplinary perspective, and find each to be at odds with evidence from anthropology and developmental science. In particular, the history of number reveals characteristics that are inconsistent with biological determinism of numerical concepts, including a lack of number systems across some human groups and remarkable variability in the form of numerical systems that do emerge. Instead, this literature highlights the importance of economic and social factors in constructing fundamentally new cognitive systems to achieve culturally specific goals.

Keywords: numerical cognition, number origins, nativism, constructivism

One exciting hypothesis about human numerical cognition is that the origins of human mathematics can be found in innate systems of quantity representation (Dehaene et al., 2008, 2009; Gallistel & Gelman, 1991, 1992; Leslie, Gelman, et al., 2008; Nieder, 2017; Szkudlarek & Brannon, 2017). Indeed, there is abundant evidence documenting an evolutionarily ancient, cross-species ability to discern discrete quantities. These abilities have been demonstrated in a wide variety of animals ranging from insects to fish, birds, monkeys (Agrillo & Bisazza, 2017; Cantlon, 2012; Jordan et al., 2005; Nieder, 2020; Pahl et al., 2013), and human infants (Feigenson et al., 2002, 2004; Jordan & Brannon, 2006; Wynn, 1992a, 1998; Xu & Spelke, 2000) even as young as 2 days old (Izard et al., 2009). Two behavioral patterns in perception have been documented (Feigenson et al., 2004; Jevons, 1871): Precise discernment of small numerosities (known as *subitizing*; with a range roughly from 1 to 4), and an approximate and ratio-sensitive discrimination ability that obeys Weber's law for larger numerosities, both of which can be derived from information-processing considerations (Cheyette & Piantadosi, 2020).

In addition to these abilities (termed "quantical" cognition by Núñez, 2017), many people also use symbolic resources for forming exact representations of large numerosities beyond the range of subitizing. This capacity can be seen in the familiar use of number words like "six hundred and forty thousand," but, as we review below, language is not the only symbolic format found in human groups. The ability to symbolically represent exact large numerosities has been argued to transcend the other quantity representation systems because, on their own, systems for representing small numerosities are exact only up to about 3–5, and the psychophysics of the large quantity discrimination system is approximate. Neither, on their own, appears able to capture many people's fluency with large symbolic numbers (Barner, 2017; Carey & Barner, 2019; Krajcsi et al., 2018; Rips, 2017; Testolin, 2020).

Here, we focus specifically on hypotheses about the origin of this system that can symbolically and exactly represent large numerosities. One foundational question in the study of numerical cognition has been to discover to what extent the conceptual knowledge for large, exact, symbolic number is innately available or biologically determined. Strongly nativist accounts provide a family of popular hypotheses to explain the origins of symbolic number. We consider theories of numerical cognition to advance "biologically deterministic" or "nativist" arguments when they posit that: (a) innate quantity mechanisms scale up, essentially on their own, to produce large, exact symbolic number and perhaps mathematics (i.e., claiming that innate quantity systems have the requisite mathematical content, rather than merely functioning to provide input to learning mechanisms); or (b) there are innate mechanisms of representation that are isomorphic to the mathematical and logical foundations of number.

Generally, in these approaches, the human-specific capacities that allow for large, exact, symbolic number mirror the elements of the abstract mathematical logic of number (for a brief overview, see Barner, 2017). Thus, these approaches suggest that the axioms

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and structures of mathematical systems like Peano arithmetic (Peano, 1890) may be part of the innate repertoire that humans are born with. While the precise sense in which these representations are innately available does not tend to be formalized, here we take it to mean that these capacities are not critically dependent on specific environmental input (i.e., they are precultural and preverbal, as with the quantity abilities we describe above that are available early in ontogeny). For instance, Gallistel and Gelman (1992) posit that an innate accumulator mechanism, following Meck and Church (1983), functions as the representational basis for not just the approximate discrimination of quantity, but also exact representations. Gelman (2004) suggests that natural numbers derive their meaning from a system of mental magnitudes which exist even before learning: "Subjects believe that the property denoted by 'three' may be added to the property denoted by 'two' to obtain the property denoted by 'five' because this is already true for the prelinguistic concepts to which the words refer and from which they derive their meaning" (p. 442). Leslie, Gelman, et al. (2008) hypothesize that human infants are born with a generative system for discrete integers consisting of an innate concept of "one" and a successor function that recursively creates concepts of higher numbers (i.e., effectively computing $S(x) = x + 1$). Although challenged elsewhere (Carey, 2009b; Relaford-Doyle & Núñez, 2018), this explicit invoking of the Peano axioms (Leslie, Gallistel, et al., 2008) leads to the strong claim that "humans possess an inbuilt learning mechanism in the form of the successor function that employs a little piece of algebra" (Leslie, Gelman, et al., 2008, p. 217). Under this hypothesis, children learning to count and use symbolic number do not discover the concepts themselves, but rather how to align the successor-system with the approximate system, and how to use language to form concise symbols for higher numerosities. Under proposals like these, the conceptual content of large exact number can be considered as biologically determined, with behaviors involving symbolic number (such as counting) being expressions of an underlying pregiven content.

In the developmental literature, strongly nativist accounts have been criticized for not explaining the empirically observed progression of stages in number learning, including the difficulty that children appear to have in learning number words (Carey, 2009a; Carey & Barner, 2019). There is strong evidence from quantitative, model-driven theory comparisons that children's early meanings are not based on an approximate system (Lee & Sarnecka, 2010, 2011; Sarnecka & Lee, 2009; Wagner et al., 2019), leaving nativist approaches that hinge on approximation unlikely to be correct. In a review of numerous training studies on approximate number, Szűcs and Myers (2017) found that studies claiming causal links between approximation and exact number had high false-positive rates and did not consider compelling alternative explanations for data, concluding that there "is no conclusive evidence that specific ANS training improves symbolic arithmetic" (p. 187).

An alternative hypothesis about the origins of symbolic number, and especially large exact number, holds that the required concepts are *constructed* out of representations that are distinct from number but, when combined appropriately, can express equivalent content (Carey, 2009a; Piantadosi et al., 2012). In this way, mathematics learning might be more like learning a skill—or collection of skills—that requires combining cognitive pieces in order to express something qualitatively different than what was antecedently available (Carey, 2009a), much like programmers create qualitatively distinct programs from simpler pieces (Piantadosi et al., 2012). More generally, the view that symbolic number is not primarily determined biologically has been advanced most recently by Núñez (2017), who argued that cross-cultural work in number has often been overlooked. Similarly, Beller et al. (2018) have argued for culture as a critically important element in the study of numerical cognition, suggesting that an interdisciplinary approach could be especially useful.

In this article, we draw on findings from both cognitive psychology and the anthropology of number in order to evaluate the position that human-like abstract number and mathematics primarily originate in biological mechanisms of quantity representation. The view that symbolic number arises through some strong form of biological determinism makes a number of predictions about the form and function of numerical systems across human groups. Specifically, if the representations at the core of symbolic number acquisition are primarily biologically determined, we should expect that: (i) number should be found in all human groups; (ii) the mode of constructing quantity representations should be universally shared; (iii) number should emerge relatively easily in development; (iv) the timing of number acquisition should be roughly the same across human groups; (v) number should emerge easily within each human group; and (vi) the history of number should exhibit the level of abstraction hypothesized in nativist theories. It is worth thinking about each of these predictions in the context of behaviors which unambiguously are biologically determined, such as puberty or some aspects of the development of vision.

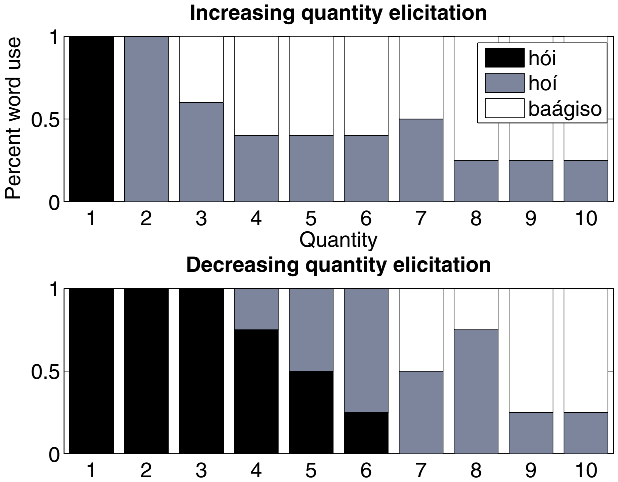
In contrast, we argue that symbolic number and counting fail all of the natural predictions (i)–(vi) of biological determinism. The cultural history of mathematics shows that, large, exact, symbolic number is cognitively difficult for humans to create, the product of a long cross-cultural history (Chrisomalis, 2009b; Damerow, 2015; Ifrah, 2000; Joseph, 1987) and that modern conceptions of number are far from universal across human groups. The picture that emerges instead from the ethnographic literature reveals a diversity of culturally contingent approaches to problems of quantity, which are constructed according to local needs, perceptions, practices, and history (Beller et al., 2018; Crossley, 2007; Dixon & Kroeber, 1907; Hymes, 1955; Owens & Lean, 2018; Robson, 2008; Wolfers, 1971). We consider each of (i)–(vi) in turn.

Cross-Cultural Absence of Exact Number

Perhaps the most striking demonstrations of cultural influence on number are found in environments that do not afford the cultural support required to create any number words at all. The Pirahã culture provides a contemporary example, with number terms being extraneous to their mode of living and thus entirely absent from the language (D. L. Everett, 2005). Tasks requiring counting cannot be solved by native Pirahã people (Frank et al., 2008; Gordon, 2004) and the only words which express quantity can be shown to mean relative, not exact, quantity. For example, when Pirahã people were asked to count 10 physical objects in ascending order (i.e., from 1 to 10), they appeared to use a one-two-many count system. That is, participants labeled sets of size "one" as *hói*, sets of size "two" as *hoí*, and all other sets as *baágiso*. However, when counting down, it was apparent that the terms were actually relative and approximate quantifiers rather than exact numerical terms—including even the word initially assumed to mean "one" (see Figure 1). Specifically,

Figure 1

Relativity of Pirahã "Number Words" (From Frank et al., 2008)



Note. See the online article for the color version of this figure.

the word that was thought to mean "one" really means "few," and it can be appropriately applied even up to 5 or 6 objects if elicitation started with 10. Similarly, the word that was thought to mean "two" really means something like "some," and the word that was thought to mean "many" does indeed mean "many." Thus the correct meanings only became apparent when the initial context was varied via the experimental manipulation of counting down. Beyond the Pirahã, these results raise the question of how many other cultures that appear to have small exact number words in actuality would be shown to have a similar relative system if the context were manipulated.

While Pirahã culture is remarkable in many other respects, other indigenous groups have used small number words in a nonexact manner. Hammarström (2010) surveys a number of languages that have been reported in the literature to have no exact number words above "one," including two languages that potentially had no exact number words at all (Oro Win and Xilixana). John Peters, a missionary-turned-sociologist who spent many years living among the Xilixana during the initial contact period, described a numerically ambiguous system that "frustrated and infuriated" him:

> Quantity is limited to three words, though the meaning can be modified by gesture. *Mõle* means one, and possibly two. *Yaluku pèk* means something between two and five, while *yalami* means anything more than two. . . . The highest amount would be indicated by using the term *yalami* together with a phrase meaning "like the trees of the forest." They once told me that there were *yalami* people in a village they had just visited. I didn't know whether the population was 16 or 80. This system obviously would not work in Western society for purchasing a bicycle or six items at the grocery store, but it was perfectly adequate for the Yanomami. Exact numbers were not important. (Peters, 1998, p. 52)

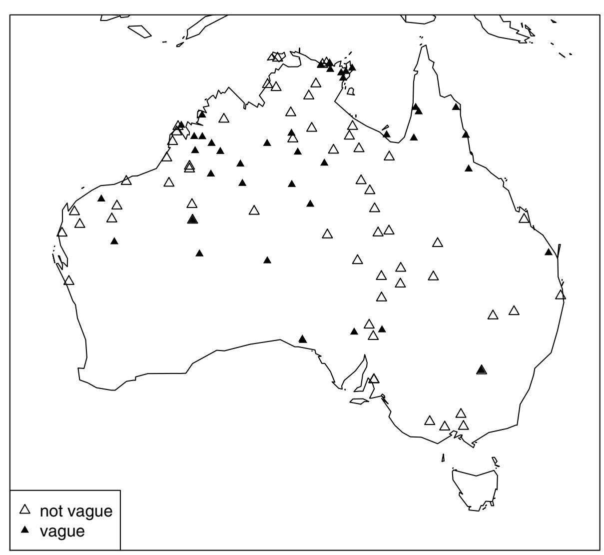
Experimental work has shown that the Mundurukú of Brazil make use of quantity terms beyond "two" in an approximate fashion, and they likewise do not verbally count: When asked to name a quantity of dots on a screen (between 1 and 15 dots), Mundurukú people mostly showed consistency with terms for "one" and "two," whereas terms for "three" and greater were not applied uniformly (Pica et al., 2004). For instance, when five dots were displayed, the term for "five" (*pũg põgbi*; or "one hand") was used only 28% of the time. Other answers included "some," "four," "three," "many," or other idiosyncratic utterances (and see Izard et al., 2008). This does not mean that quantity judgments are impossible for Mundurukú speakers, as exact small quantity and larger approximate quantity estimations seem to be universal and are easily demonstrated in the absence of learned number symbols (Butterworth et al., 2008; Dehaene et al., 2008; Frank et al., 2008, 2012; Pica et al., 2004). However, recent work by C. Everett (2019) has argued that even small numbers are not generally privileged in lexical systems of the world.

The use of numerical terms in a nonexact or vague manner has also been documented for a number of Australian indigenous groups (see Figure 2), including one language where the term for "one" could be used approximately (Warlmanpa; Bowern & Zentz, 2012). The existence of vague number words in Australia suggests a lack of utility for precise numeration prior to colonization (Harris, 1982), although small numeral systems do not necessarily imply vague number usage (Bowern & Zentz, 2012). Yet as McGregor (2004) points out, in cases where collections with numerosities exceeding subitizing range are culturally designated as "many," it makes little sense to apply an exact counting procedure since the numerically labeled quantities can be immediately and nonverbally apprehended.

Such cultural patterns would be surprising under theories in which all humans have an innate concept of "one" along with an innate successor function, since the concepts that these systems generate are not lexically marked in languages like these, but nearby quantity terms (e.g., "a few") are. While the potential lack of utility for exact quantification historically might be invoked to defend strongly nativist accounts, such reasoning begs the question of why symbolic number (and especially large number) would be biologically encoded in the first place if it were not useful.

Figure 2

Vague and Nonvague Australian Numeral Systems (From Bowern & Zentz, 2012)



Pirahã provides an interesting case in this context, where it is likely that symbolic number would be useful in trade, and yet it is neither adopted nor constructed. In fact, Pirahãs rejected number words and counting as outsider knowledge which is not required for a satisfying life (D. L. Everett, 2005). Explaining these cultural patterns remains challenging under strongly nativist accounts.

Forms of Number Representation Are Diverse Across Cultures

Many nonindustrialized people have developed rich numerical systems that are quite different from our familiar decimal counting system. Binary, quinary (base-5), and vigesimal (base-20) systems have commonly been encountered (Flegg, 1983/2002; Menninger, 1969/1992), along with comparatively rarer forms such ternary, quaternary, senary (base-6), and others (Hammarström, 2010). But interestingly—and perhaps counter-intuitively—the concept of a single counting base is not sufficient for explaining the variety of systems that have been invented, with body-part systems and multiple bases also being common, thus necessitating new forms of classification (Hymes, 1955; Laycock, 1975). Body-part systems have been observed in South Eastern Australia (Howitt, 1904, pp. 697–698), and the Torres Strait Islands (Ray, 1907), with Ray (p. 47) describing a base-2 verbal count coexisting with a 19-part body tally (which started on the little finger of the left side and looped over the body to the little finger on the right hand). The names for the places were distinct from the numeral roots for "one" (*urapun*) and "two" (*ukasar*) however, as they were instead the literal names of positions on the body. Similar body-part systems have been documented in Papua New Guinea (Franklin & Franklin, 1962; Saxe, 2014; Williams, 1940b). Verbally these systems can be compared with modulus systems, although they may function like base counting systems when totals are able to be carried, either mentally or with additional bodies (Wolfers, 1971). An abundance of multibase systems have also been studied in Papua New Guinea and Oceania (Owens, 2001; Owens & Lean, 2018), for instance the common 2-5-20 cycle, where counting proceeds: 1, 2, $2 + 1$, $2 + 2$, 5 (often one "hand"), $5 + 1$, $5 + 2$, $5 + 2 + 1$, $5 + 2 + 2$, $5 + 5$, and so on, up to a new base of 20 (often one "man"). Even systems of finger counting show a great deal of diversity (Bender & Beller, 2012), as do numerical notation systems (Chrisomalis, 2009b).

While it may seem that the morphological labeling system for integers is a degree of freedom removed from the underlying semantics of a discrete infinity of integer concepts, in some cases morphological patterns seem inconsistent with notions of an innate recursive number generator. For example, the theory of an innate successor function posits that each natural number is generated as the successor of a previous natural number, with recursion grounding out at "one" (Leslie, Gelman, et al., 2008). This would suggest that addition by one—application of the successor function—should provide the most natural base system for counting since it would transparently map between words/morphology and meaning. However, even the way that count series are linguistically formed varies beyond the additive, as some systems exhibit multiplication and even subtraction in their construction (Bowern & Zentz, 2012; Carrier, 1981; Dixon & Kroeber, 1907).

Yoruba counting is one well-known system that employs subtraction, with 15 root words and a primarily vigesimal structure (Verran, 2000). An example illustrating subtraction can be seen from the numeral words for 40–60, which in decimal form could be translated as: 20×2 [forty], $1 + (20 \times 2)$ [forty-one], $2 + (20 \times 2)$ [forty-two], and so on to forty-four, followed then by: $-5 - 10 + (20 \times 3)$ [forty-five], $-4 - 10\ (20 \times 3)$ [forty-six], et cetera up until: $-10 + (20 \times 3)$ [fifty]. To give a written example in Yoruba, "forty-five" would be *márùúndínláàádọ́ta;* where "twenty placed three ways" (the elision *ọ́ta*) occurs at the end of the phrase. Prior to this we have *m* (mode grouped), *árùún* (mode 5), followed by the elisions *dín* (it reduces), and then *láàád* (add 10 it diminishes; see Verran, 2000, p. 350). From fifty-one to sixty the system proceeds: $1 - 10 + (20 \times 3)$ until fifty-four, then followed by: $-5 + (20 \times 3)$ and so on until sixty, or 20×3. According to Mann (1887), this particularly complex system may have a cultural genesis in the counting and distribution of cowrie shell currency, and in her earlier work Verran (2000) noted the etymological connections to the counter's hands and feet.

Variation in form goes beyond the structure of the counting system and includes its use. This may seem unfamiliar because, in cultures with formal education, we may consider the ability to apply numbers to any set as one of their defining features. However, some cultures have imposed constraints on the category of items that their number words can be applied to (Seidenberg, 1962), suggesting that the generalizability and abstraction that our culture finds in number—and encodes into features like an innate successor function—does not generalize well to other human groups. Such diversity is particularly evident in Papua New Guinea (Wolfers, 1971), where variations likely developed through a combination of innovation and diffusion, moderated by cultural constraints on necessity and interest in enumeration (Owens & Lean, 2018). For example, Ponam Islanders have an extensive decimal count system with terms upwards of 9,000, yet strikingly, not everything would be counted with this system. Carrier (1981) reported that as a rule, they did not count people:

> Despite obvious skill with numbers, no one has any idea how many people live on the island, how many households there are or how many children are attending the primary school. Even more surprising, many parents of large families do not know how many children they have without stopping to think about it. And almost no one knows that there are 14 clans on the island, although everyone knows their names and can calculate the number in a few moments. (p. 471)

Saxe (2014) has shown how the introduction of monetary exchange drives the creation of new forms of number representation and arithmetical abilities, but exchange itself does not necessarily imply number usage. For some groups the exact number of gifts given in a ceremonial exchange is important (e.g., the Ekagi or the Melpa; Owens & Lean, 2018, pp. 132–133), whereas for others, it is the visual quality of the presentation that takes precedence, with exact number playing little to no role (e.g., the Adzera, Dani, or Loboda; Owens & Lean, 2018, pp. 128–132). For the Ponam, it is not the absolute number of objects that matters in a ceremonial exchange, but rather the number relative to another's gift (Carrier, 1981).

Symbolic Number Learning Is a Difficult Developmental Process

Ordinary language learning is notable for its rapidity and richness. Children begin to use their first words within about a year

(Brown, 1973) and can acquire word meanings from even single instances (Carey & Bartlett, 1978; Heibeck & Markman, 1987; Spiegel & Halberda, 2011). Analyses of the distribution of ages at which children learn words have even suggested that across languages, children typically require only about 10 examples in order to learn a word, and that these most useful examples come relatively independent of a word's overall frequency (Mollica & Piantadosi, 2017). In comparison, acquiring the correct numerical meaning of number words (i.e., not just the words themselves) is a lengthy and difficult process (Baroody, 2006; Carey, 2009a; Carey & Barner, 2019). Children in the U.S., Japan, and Russia typically learn meanings for number words between ages 2.5 and 4 years (Le Corre & Carey, 2007; Lee & Sarnecka, 2010, 2011; Piantadosi et al., 2014; Sarnecka & Lee, 2009; Wynn, 1990, 1992b), even though they know how to recite these words in sequence before the process even begins (Carey, 2009a; Fuson, 1988; Gelman & Gallistel, 1986). Why is it, despite being in a culture that strongly values numeracy and teaches it with omnipresent stimuli like Sesame Street and counting board games, that U.S. children require over a year to learn the meanings involved in number? Moreover, even learning to count properly does not appear to be fully sufficient for generalizing children's number knowledge to all possible number words (Davidson et al., 2012).

The stages of learning are suggestive of genuine conceptual creation (Carey, 2009a), where children arrive at an insight into how a counting procedure relates to cardinality. In line with this, detailed modeling and experimental work has shown that children's meanings do not appear to rely on approximate estimation—which is shared with other species and available in infancy—but instead follow a stage-like progression before a sudden jump in understanding (Lee & Sarnecka, 2010; Sarnecka & Lee, 2009; Wynn, 1990, 1992b). This stage-like progression can be explained by learners who do statistical inference over a space of *procedures* themselves (Piantadosi et al., 2012). More recent work has shown how learners might construct generative mental theories of entire structures like the integers (infinite, ordered, and discrete) from a simpler basis that does not presuppose this conceptual structure, but is able to acquire many different structures across domains (Piantadosi, 2021). This general approach of learning procedures and representations is notable in drawing on inferential processes that have been argued for independently in concept learning (Amalric et al., 2017; Calvo & Symons, 2014; Depeweg et al., 2018; Erdogan et al., 2015; Goodman et al., 2008; Goodman et al., 2015; Lake et al., 2017; Piantadosi & Jacobs, 2016; Romano et al., 2018; Rothe et al., 2017; Rothe et al., 2016; Wang et al., 2019; Yildirim & Jacobs, 2015), potentially showing how number systems may be constructed like other—even artificial—systems of rules that adults acquire and fluidly manipulate.

Importantly, even as children learn number, they seem not to automatically understand what we would consider principles of arithmetic (cf., Gelman & Gallistel, 1986). Children are able to apply the counting procedure before they have an explicit understanding of a successor function (Cheung et al., 2017), or other properties like commutativity (Baroody & Gannon, 1984) and infinity (Cheung et al., 2017). Such a protracted period of learning suggests that acquiring these principles is not driven by initial conceptual knowledge of axiomatic properties of number (Carey Barner, 2019).

The Timing of Symbolic Number Acquisition Varies Cross-Culturally

While there is certainly cultural variability in the age at which biologically determined events occur (e.g., in motor development; Karasik et al., 2010), the picture from number learning is starkly different in scale: The age at which children learn the meaning of number words appears to be driven almost entirely by external factors. Evidence for this comes from some of our own work with the Tsimane', an autarkic indigenous people from lowland Bolivia. The Tsimane' language has a decimal system of numeration, although it is possible that a quinary system was formerly employed (Sakel, 2011, pp. 167–168). While counting and number have grown in importance for Tsimane' people over several decades (e.g., the use of Spanish has increased mainly due to market contact and government education initiatives), mathematical abilities still remain of marginal importance when considered relative to large-scale industrialized societies.

Tsimane' children proceed through the same stages of number learning observed in the U.S. but at a much slower rate, sometimes taking three to four times as long to acquire counting (Piantadosi et al., 2014). For comparison, we know of no variation in biology that is changed by a factor of 3—it would be as though children in the U.S. went through puberty in their early teens, but in some cultures this happened around age 40. This argues strongly against, for instance, biological maturation as the cause of children's difficulty in number learning. Instead, the timing of number acquisition for Tsimane' is likely caused by being exposed to far fewer formal and informal instances of number usage, including much less child-directed speech in general (Cristia et al., 2017). This timing difference is unlikely to be purely about learning the words themselves (Boni et al., 2021), as the acquisition curves in Tsimane' number are most consistent with them needing to wait longer to observe the required data for learning or constructing number (Mollica, 2019). These effects mirror in an indigenous group known effects of input on number acquisition even in the U.S. and the Oksapmin (Saxe, 1981). The implication then is not only that verbal counting systems are cognitively difficult to develop, but that they crucially depend on a great deal of learning and input—the extent and nature of which in turn hinge on the cultural context.

A data-driven hypothesis matches studies in the U.S. which have found a strong effect of the amount of data learners receive in driving their acquisition (Levine et al., 2010). In some cases, the relevant data might be subtle and depend on other representational systems. For instance, Almoammer et al. (2013) found that the timing of Slovenian and Saudi Arabic number acquisition is likely influenced by the presence of dual grammatical markers in these languages. In both cases, almost total dependence on data is consistent with the lack of symbolic number in some cultures [see (i) above], and hard to understand under theories that derive number primarily from biology rather than primarily from experience.

Long Historical Development of Symbolic Number

The development of symbolic number concepts throughout history has been neither uniform nor universally shared across different contexts. While all humans appear able to perceive quantity and have the ability to develop symbolic number and counting given the appropriate input, the extent to which exact number concepts are actually developed is driven by environmental and cultural factors. For example, Divale (1999) has argued that climatic conditions predict the development of counting systems in traditional societies (in terms of highest count), specifically through situations where food storage and calculation are important for survival. This suggests that counting systems in such situations are developed as solutions to culturally relevant problems.

However, the relation between number and culture is not found in a simple unilineal social development of number concepts (Donohue, 2008), as some 19th-century theorists believed (e.g., Crawfurd, 1863). Epps et al. (2012) for instance have argued that hunter-gatherer and agriculturalist modes of social organization are not good predictors of number system development, and in the development of written numerical systems, Chrisomalis (2009b) provides evidence for a multilineal, rather than unilineal evolution. Yet the greater elaboration of number systems is, in general, related to greater sociocultural complexity (Chrisomalis, 2009b; Divale, 1999; Nissen et al., 1993; Overmann, 2016; Robson, 2008; Schmandt-Besserat, 1978). Sociocultural complexity here refers largely to matters of material culture, hierarchical division, socioeconomic stratification, population density, and the production and management of surplus (e.g., for ceremonial feasting; Hayden & Villeneuve, 2011). Large-scale comparative ethnological analyses suggest that such complexity precedes the greater elaboration of number systems (Overmann, 2016), and hence number development can be seen as a class of pragmatic solutions to social exigencies—exigencies which have not been universally shared across cultures throughout history (Hayden & Villeneuve, 2011; Saxe, 2014).

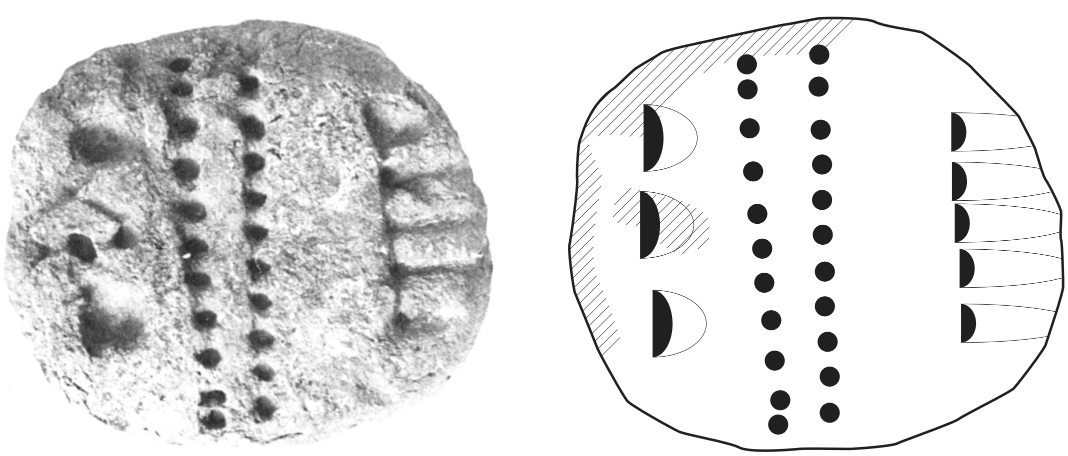
The suggestion that material and complex cultural needs provided the impetus and cognitive scaffolding for number development (Overmann, 2013) fits with archeological evidence demonstrating the historical development of abstract number systems. The history of numbers in the ancient Near East, which effectively is the early history of writing and accounting, provides a striking and well-attested example of the social roots of number. Toward the end of the third millennium BCE in Mesopotamia, a numerical system akin to modern numerals was developed using cuneiform script (Damerow, 2015; Damerow & Englund, 1993b); however, the entire cuneiform writing system had its genesis in a much earlier bureaucratic need for administrative control. Over the span of thousands of years, a variety of quantitative control mechanisms emerged that were in fact the pragmatic responses of a centralized bureaucracy to growing economic complexity. In particular, these mechanisms consisted of increasingly refined techniques for information recording and quantification (Nissen et al., 1993, p. 116).

The use of physical material (such as stone and clay tokens) to track goods stretches back for millennia before the oldest known written signs were made on tablets, first occurring somewhere around 3,400 BCE (Friberg, 1994; Schmandt-Besserat, 1986). The contents of these tablets consisted exclusively of numerical content (Englund, 2004). However, unlike later texts, the very earliest written symbols had not yet been codified into a well-defined system. The early symbols were likely linked to prior counting practices using clay tokens (e.g., some symbols are clearly impressed tokens; Englund, 2004; Friberg, 1994; Nissen, 1993), but they were used in a flexible manner and probably document a transitional phase to a more structured numerical system (Damerow & Englund, 1993b). For example, at this stage signs could apparently be repeated an arbitrary number of times, in comparison to the decimal limit that appears later (e.g., see Figure 3).

The practical deficiencies of such a flexible system likely led to the emergence of protocuneiform in archaic tablets sometime around 3,100 BCE. Used primarily for accounting purposes, the numerical content of these tablets was highly structured and rule-based, but the symbols were not representations of a single abstract system of natural numbers. Instead, the texts indicate a variety of pragmatic and context-dependent metrological systems, the symbols of which expressed both quantitative and qualitative information and which were therefore unlike modern numerals—a fact which eluded Assyriologists for many decades (see Damerow & Englund, 1993a; Englund, 2004). For instance, depending on the metrological context, the same ● symbol could denote different amounts of various goods. Thus unlike a numeral, a single ● could represent 10 sheep in one system, 6 lots of barley in another

Figure 3

Photograph (van Driel, 1982) and Line Drawing (Englund, 1998) of an Early Simple Numerical Tablet with Non-Standard Numerical Grouping (Jebel Aruda, ca. 3500–3350 BCE)



(about 150 L), or 18 lots of field area in yet another system (about 6 ha; Nissen et al., 1993, pp. 25–29, pp. 131–132), among other possibilities.

It was not until the end of the third millennium BCE, centuries after the appearance of the proto-cuneiform texts, that a "pure" number system was developed in the form of the sexagesimal place value notation system. Being a universally applicable symbolic system that expressed only the abstract concept of number, this new system was initially used to translate values between the various metrological systems, and according to Robson (2008) it "temporarily changed the status of numbers from properties of real-world objects to independent entities that could be manipulated without regard to absolute value or metrological system" (p. 78). Unlike those earlier numerical systems which proliferated symbols fit for specific metrological contexts, the new symbols were akin to modern numerals, although they operated on base-60 (with 10 as a subbase) and without a symbol for zero. To return to the earlier example with the context-dependent ● symbol, by now 1, 6, 10, or 18 units of anything could be expressed as 𒁹, 𒐋, 𒌋, and 𒌋𒐍, respectively. While this system did not supplant the traditional systems for accounting purposes, it enabled mathematical investigations that were of little practical value administratively, and found application some 1,500 years later in Babylonian astronomy, perhaps the earliest empirical science that can be historically linked to modern practice (Damerow & Englund, 1993b; Joseph, 1987; Pingree, 1992).

This record suggests that the very idea of a natural number (in the mathematical sense), or the creation of a symbolic system that strictly expressed that concept, was a drawn-out process in ancient Mesopotamia, being dependent on particular sociopolitical conditions and needs. The implication is not that prior to these developments Mesopotamian people had no numerical concepts nor that this history represents a universal cultural evolutionary stage (and see Chrisomalis, 2009a), but rather that particular social conditions were required for the development of these symbolic systems which were important in enabling further conceptual developments in number and mathematics (Joseph, 1987). For example, the positional principle of the sexagesimal place value system used in Babylonian astronomy was later adopted by the Greeks, who appear to have merged it with their own alphabetic numeral system (Chrisomalis, 2009b). The constructed nature of the "integer" then seems to run parallel to another mathematical invention—negative numbers. The concept of debt in accounting was known and utilized in Mesopotamia without the concept of negative numbers (Nissen et al., 1993), and such numbers were resisted in Europe for quite some time (Mattessich, 1998). Yet debt obviously can be conceptualized as a negative quantity, and the first full mathematical acceptance of negative numbers, about 600 CE in India, was likely derived from accounting practices (Mattessich, 1998). This again illustrates that aspects of number which today may seem natural or fundamental, were derived culturally in response to specific material needs—not axiomatically from biology.

Large Exact Number Representations Started Concretely, Not Abstractly

Strongly nativist theories assume that the key abstractions—high-level knowledge about integer concepts, like a successor function, that is not dependent on a particular physicalization—are innately specified. However, the anthropological history of number suggests that early systems for quantity representation drew on concrete physical objects, not on preexisting abstract concepts of integers or successorship. Indeed, the history of number charts that abstractions only arose when social conditions necessitated increasingly complex quantitative solutions.

In the early ethnology of indigenous approaches to number, much was made of the preponderance of concrete referents and techniques (Conant, 1896).[1] It was noted that number words frequently had body-part referents (e.g., *hand* for "five"), and also that external material such as counting sticks or tally objects were often utilized, demonstrating a dependence on physical items that would be odd if the required concepts were shared innately. Such external materials were often used alongside or in place of a developed verbal counting system. This remarkable phenomenon—marrying physical and verbal—has been almost entirely lost in psychological discussions of how humans represent number (but see Bender & Beller, 2012). For example, Fortune (1942, pp. 58–60) reported that the Mountain Arapesh counted without difficulty using only three numeral root words. Each time their count exceeded 24, they would stake a physical peg into the ground to keep track of how many multiples of 24 had been passed. Thus, this system represented exact quantities modulo 24 in a verbal system, and multiples of 24 with physical stakes. In Papua New Guinea, counts that surpassed the limit of a body-part system would necessitate another physical body if the ability to mentally carry the total had not been developed, as it had not for some groups (Wolfers, 1971). There is also evidence from Southern New Guinea of item-specific counting systems arising from physical practices, for instance in the counting of yams (Evans, 2009; Hammarström, 2009).

The use of external material could also enable exact quantity manipulations outside of a verbal counting system. For example, Iqwaye people historically managed quantities that were likely beyond the practical limits of their count system: Mimica (1992, p. 16) describes a knotted rope with 161 shells historically used for enumerating warriors, where rather than verbally counting individuals, men could instead be marked off against each shell one by one to obtain the proper amount. In a similar manner, some Australian aboriginal people notched message sticks as mnemonics without necessarily needing (separate) symbolic number: Each mark would simply correspond to some informational item that the messenger needed to relay (Howitt, 1904, pp. 691–710). Howitt (1904) also described the estimation of time whereby a messenger made, or in other cases removed, a body mark for each day traveled (p. 702). While it is known that aboriginal Australian counting systems were diverse and not uniformly one-two-many systems (Bowern & Zentz, 2012; Dawson, 1881; Harris, 1982), such techniques could still be utilized among those groups that did not develop extensive verbal counting systems.

As McGregor (2004) points out, these methods of tallying and marking are conceptually related to counting, yet they remain symbolically and cognitively distinct activities. In Peirce's classification

[1] Many of these early authors were influenced by 19th century notions of social progress (e.g., Conant, 1905; Crawfurd, 1863; McGee, 1899), and attributed numerical differences to racial and social inferiority (cf., Dawson, 1881). Critiques can be found in Harris (1982) and Mimica (1992), or Verran (2000), and tangentially in Donohue (2008) and D. L. Everett (2005, 2009), and McGregor (2004).

(Burks, 1949), a number word and a tally mark both constitute signs, but the number word is *symbolic* in that it arbitrarily points to some numerosity, whereas the tally set is *iconic* in that it always contains its own referent (i.e., its own numerosity in marks). What is shared by these systems then appears to be a capacity for instantiating one-to-one correspondence between sensory objects and linguistic or physical markers, suggesting that a principle like one-to-one correspondence is a necessary precursor to number (Carey & Barner, 2019; Izard et al., 2009), particularly given that it is a strategy used by children before learning number (Jara-Ettinger et al., 2015). However, young children initially have only a partial understanding of one-to-one correspondence as it relates to exact number, and they cannot relate the principle to arbitrary set sizes (Izard et al., 2014; Muldoon et al., 2005). Moreover, capacities suggesting a potential for understanding one-to-one correspondence have recently been found in nonhuman primates (Koopman et al., 2019), again raising the challenge of what specifically differs between humans and other species as a key issue for the field.

If one-to-one correspondence is necessary in order to construct number, that makes the natural prediction that one-to-one correspondence tasks should be possible even within groups that lack symbolic number. Among the Pirahã results have been mixed, with Gordon (2004) and C. Everett and Madora (2012) reporting failures on one-to-one matching in the absence of number words. Frank et al. (2008) on the other hand were able to replicate Gordon's findings, but they showed success on the one-to-one matching task. In the Frank et al. (2008) study, Pirahã people were asked to line up markers one-to-one with a line of items provided by an experimenter. When objects were placed in parallel, making the one-to-one alignment natural, participants were able to match items one-to-one. However, when asked to match with an orthogonally presented array, where a visual strategy of verifying correspondence without number words becomes very difficult, they did not succeed (see Figure 4). In a similar manner, the use of one-to-one matching in dot counting tasks by some Mundurukú people has been reported by Pica et al. (2004), who noted that some participants (when prompted) were able to "count very slowly and non-verbally by matching their fingers and toes to the set of dots" (p. 500). This in turn suggests that one of the primary utilities for symbolic number is in providing a verbal memory cue for one-to-one correspondence. Indeed, Frank et al. (2012) showed that English undergraduates' performance in matching tasks mirrors those of Pirahãs when they are given a dual task to prevent verbalizing. In turn, the earliest archeological evidence for tally systems reflects iconic one-to-one correspondence, with artifacts appearing to demonstrate repetitions of single units like ||||| (Bogoshi et al., 1987; Coolidge & Overmann, 2012; d'Errico et al., 2012), as opposed say to cross-tallied unit groupings like 𝍸.

Of course, the archeological record depends exclusively on found physical material, and it is not easy to draw firm conclusions about the underlying meanings (although elaborate attempts have been made; Elkins, 1996; Marshack, 1964; Robinson, 1992). Moreover, the history of nonartifactual and potentially ancient counting techniques such as finger counting (Bender & Beller, 2012), and even numeral words, are not fully recoverable. Yet by turning to historical linguistics we see that the oldest known verbal counting systems, likely the two-cycle systems (Owens & Lean, 2018), are constructed out of numerical roots for numerosities within subitizing range only (i.e., 1 and 2), again pointing to the role of external material and genuine conceptual construction in creating higher number representations (Overmann, 2018). The unusual historical stability of low-limit number words, in contrast to higher number words (Pagel & Meade, 2018), also suggests a primary role for perception (Barner, 2017).

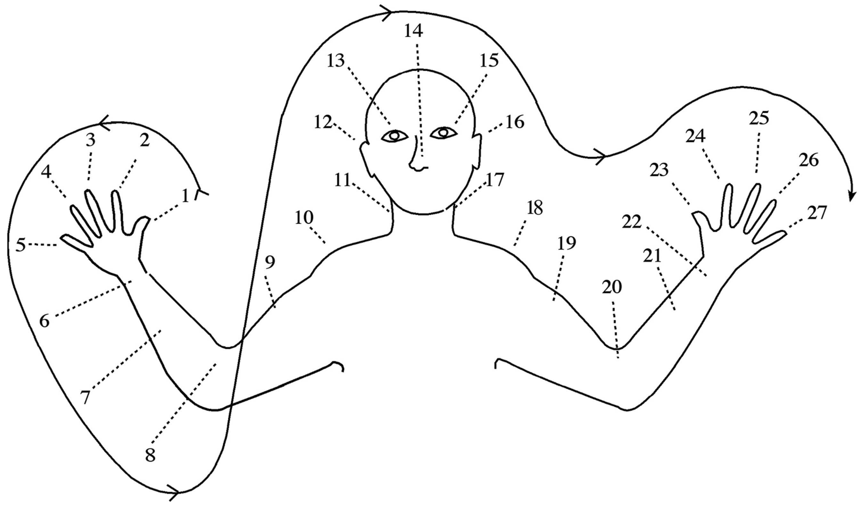
Body-part systems can present a bridge between externalized and verbal counting, particularly when they function physically like tallies yet have named positions like a verbal count list (e.g., see Williams, 1940a). Saxe (2014) carried out a series of numerical cognition studies with the Oksapmin in 1980, who at the time utilized a 27-part system (see Figure 5), in addition to a separate list of conversational and noncounting words for numbers up to "five" (Saxe, 2014, p. 331). When presented with an addition task with objects present (e.g., six objects plus eight more), elders had no difficulty enumerating the total set, and indeed many would physically touch each individual object once along the body system, eventually exhausting all objects and thereby arriving at the correct sum. However, when the objects were not visible, the same task proved difficult. On the other hand, many

Figure 4
Line-Match Tasks Used by Frank et al. (2008)

(a) *Parallel match.* (b) *Orthogonal match.*

Note. See the online article for the color version of this figure.

Figure 5
Oksapmin Body-Part System (From Saxe & Esmonde, 2005)



younger Oksapmin people succeeded via the use of novel strategies; for instance by symbolically dissociating position names from their usual physical location (see Study 4-1; Saxe, 2014). Such strategies were more common and elaborate among those with greater participation in the expanding market economy, demonstrating that even seemingly simple mental arithmetic requires cultural scaffolding in order to be developed. In effect, purely mental arithmetical operations were not fully available for the elders, even well-within range of the body-part system, simply because the supporting cognitive strategies had not been widely developed (presumably by virtue of their lack of relevance in traditional Oksapmin life; Saxe, 2014, p. 47). It should also be noted that symbolic abstraction in body-based systems such as finger counting is widespread and diverse (Bender & Beller, 2012), again pointing to a culturally contingent interplay between mind and body. More recently, the extent to which concrete methods for manipulating number can be internalized has been demonstrated dramatically by the practice of mental abacus, although even then the role of gesture remains critically important (Brooks et al., 2017).

The idea that abstract number systems have their roots in physical material also finds parallel in the historical development of measurement systems. In the same way that the etymology of English language measurement words show a strong bias toward the body or external objects, small-scale societies have had measurement terms that are "overwhelmingly based on the body" (Cooperrider & Gentner, 2019, p. 3). The historical process of abstraction that occurs more generally in materially complex societies relies upon reflection and the construction of new concepts which arise from material structures acting as cognitive scaffolding—a process in which tallies are but an early example (Overmann, 2016). Yet as Cooperrider and Gentner (2019) point out, there is no straightforward path toward greater abstraction (and see Chrisomalis, 2009b; Donohue, 2008). Based on analysis of the historical and ethnographic record of measurement practices, they conclude that the use of abstract units is not a natural or intuitive activity, but rather is a culturally evolved practice that develops over long periods of time. For instance, just as the Adzera of Papua New Guinea would keep records of gifts to be reciprocated using string bark tallies (Owens & Lean, 2018, p. 129), the practice of recording debts on stocks of notched wood survived at the English exchequer until the 19th century (Baxter, 1989; "Records By Tally Sticks," 1909).

Concluding Discussion

Because we live in highly numerate and literate societies where virtually every adult understands not just counting but also at least some higher mathematics, it is easy to lose sight of the fact that throughout human history there has been a striking diversity of ways to represent number. Within the field of numerical cognition, the prevalence of theories based in biological determinism may be due to the fact that much of the work on number has prioritized animal work over data from nonindustrialized cultures (Núñez, 2017). Animal work has been instrumental in understanding fundamental mechanisms of quantity representation across species, but nonhuman animals do not learn symbolic number in the same way as human children, nor do they have anything approaching the breadth of other mathematical abilities that are available to humans. Thus, in seeking the origins of number, it may well be that the differences between humans and animals ought to be emphasized, as opposed to the similarities.

A richly accumulative culture may be one of the primary differences between animals and all human groups. Depending on powerful and general learning mechanisms, cultural transmission permits mathematical knowledge to be passed down and perhaps leads to striking cascades in how people think (C. Everett, 2017). Formal and informal pedagogical practices prevent each generation from needing to reinvent mathematical abstractions (revising old ones instead), in what Tomasello (2009) called "the ratchet effect." If part of this ratchet works to simplify and abstract, then over time cultural transmission may tend to remove complications that are otherwise perfectly natural for humans—such as counting systems that depend on the type of object, rely on subtraction, interface with physical objects, or use icons. If scientists happen to live in a culture where this ratchet has led to a particularly simple and abstract formulation of numbers (e.g., the Peano axioms), it would be easy to confuse the outcome of this cultural process with human nature—especially if the breadth of human numerical creation was not well appreciated. It is easy to forget, for instance, that the mathematics we know today—including concepts as simple as fractions, real numbers, and zero—has been unknown throughout almost all of human history, even up through the scientific revolution (e.g., Grattan-Guinness, 1996; Ifrah, 2000; Kleiner, 1988; Logan, 1979; Mattessich, 1998). Galileo formalized his physics with geometrical arguments, not algebraic equations, but generations of successive physicists have been able to reformulate the underlying ideas of physics in increasingly general and simple mathematical forms, from Newton's laws, to Lagrangian and Hamiltonian mechanics, and more. Just as it would be a mistake to suppose that modern, elegant incarnations of physics must be innate because they are simple, it equally would be mistaken to think that distinctly WEIRD (Henrich et al., 2010) formulations of natural number somehow reflect a universal human nature.

Instead, the early ethnographic history suggests that representations are constructed ad hoc according to the cultural stock of available techniques—which are not necessarily attached to an underlying integer-like representation. Even when two domains are both plausibly numerical, some cultures have invented separate quantity systems which are not necessarily easily translatable. For the Kewa of Papua New Guinea, Franklin and Franklin (1962) write of a division between verbal counting and a body-part system used for specific purposes such as calendar reckoning: "The body-part system is not usually used to specify an exact number, e.g., one, five or ten. Informants cannot give the body-part system equivalent of a four base system number. Instead, the words meaning a few, lots, several, are used" (p. 189). In terms of verbal counting, some indigenous groups have historically used count systems with relatively high practical upper limits, some have only linguistically marked numerosities of small sets within subitizing range, and others have not marked exact numerosities at all (at times excluding even an unambiguous numerical term for "one"). Iconic techniques, based on one-to-one correspondence, have allowed some groups to solve quantitative problems without the use of the symbolic number. The movement to mathematics in the current Western sense, and hence the study of numbers as abstract entities unto themselves, was dependent on the development of writing and the concerns of complex hierarchical state-like societies, and hardly a natural or universal occurrence. Indeed, Chrisomalis (2009b, p. 427) argues that the most significant event in the history of numerical notation was not the often-cited invention of positional systems or the discovery of zero, but in fact the rise of capitalism as a dominant world system.

Importantly, our claim here is not that evolution and biology play no role in symbolic number learning—clearly humans differ from other animals in their capacity for mathematical and numerical cognition, and this difference could not exist without a biological foundation. As in any other cognitive domain, comparison between humans and other animals is likely to be informative (e.g., Cantlon, 2012), for instance, about how evolutionarily ancient systems shape our own learning and perception. Our claim is that whatever biological heritage is relevant for large exact numbers, the biology does not determine the form of mental numerical content. We have argued that this is evidenced in the diversity of numerical systems that exist, the cultures in which they do not, and the difficulties of constructing number both historically and developmentally. The history of number shows that appeals to nativism—specifically innate content tantamount to the integers—oversimplify and obscure crucial sociocultural processes. Thus, although our focus here has been on cultural factors, it is apparent that the correct picture is in fact a biocultural one. It is likely that biological learning mechanisms can be broadly deployed to learn structures and procedures across domains, supporting the variety of numerical forms that have emerged cross-culturally and throughout history (as well as their absence).

This points to the need for developmentalists to pursue theories of how procedural knowledge may be learned, represented, and taught (Rule et al., 2020); perhaps drawing on our ability for metaphor (Lakoff Núñez, 2000; Núñez, 2009) and structure more generally (Tenenbaum et al., 2011). One approach to this is to formalize statistical learning theories that work over general spaces of algorithms like those in counting and other domains because children learn counting as an algorithm. Indeed, developmental theories of early number should have drawn on the now classic educational literature showing that children have a rich ability to revise and improve algorithms that they have been taught in domains like addition (Ashcraft, 1982, 1987; Groen & Resnick, 1977; Kaye et al., 1986; Siegler & Jenkins, 1989; Svenson, 1975). Models of such learning argue that children effectively choose between known procedures (Siegler & Jenkins, 1989; Siegler & Shipley, 1995; Siegler & Shrager, 1984) or create fundamentally new procedures (Jones & Van Lehn, 1994; Neches, 1987). If such abilities are available early in learning, and people's capacity to infer such procedures extends far beyond numerical content, it is then most natural to hypothesize that number learning itself depends on these much more general algorithmic mechanisms of learning and representation (Piantadosi, 2021).

Nativist developmental theories have tended to argue that specific innate content is required for large exact representations, while ignoring the fact that developmental processes can acquire so much more—including fractions, real numbers, complex numbers, vectors, matrices, tensors, arbitrary groups or fields, transfinite numbers, logic, set theory, calculus, non-Euclidean geometry, computability theory, and so on. Considering the breadth of human learning ability, theories which posit that exact symbolic number is determined by innate resources—while accepting these other domains as constructed—would seem to miss the marks of parsimony and adequacy, in addition to failing the most natural empirical predictions we describe above.

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