Bayes Rule

Intuitive Description of Bayes

Initial belief plus new evidence = new and improved belief.

$$P(H|D) = \frac{P(H)*P(D|H)}{P(D)}$$

Bayes Rule Components

Hypotheses – beliefs you have about the world

Probabilities – strength of belief that each hypothesis is true

Priors – assumptions about the world (before you see data)

Likelihoods – your assumptions about how the data you see was generated by hypotheses

Steps in computing Bayes

1. Compute the prior for each hypothesis, P(H)

2. Compute the likelihood of the data under each hypothesis, P(D|H)

3. Multiply prior times likelihood to get P(H) P(D | H)

4. Re-normalize these over all hypotheses (e.g. scale each P(H) P(D|H) value so that together they sum to 1; equivalently, divide each by the sum of all of them).

Plug-in Example:

Cancer A is estimated to occur in one percent of people your age.

Test A is 98 percent reliable.

- e.g., 98 out of 100 people who have cancer will test positive, and 98 out of 100 who are healthy will test negative.

Your test is positive. How probable is that you have cancer?

Modified from: http://blogs.scientificamerican.com/cross-check/bayes-s-theorem-what-s-the-big-deal/

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Hypothesis	P(H)	P(D|H)	P(H)P(D|H)	P(H|D)
You have Cancer				
You don’t have Cancer				

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Hypothesis	P(H)	P(D|H)	P(H)P(D|H)	P(H|D)
You have Cancer	0.01			
You don’t have Cancer	0.99			

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Hypothesis	P(H)	P(D|H)	P(H)P(D|H)	P(H|D)
You have Cancer	0.01	0.98		
You don't have Cancer	0.99	0.02		

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Hypothesis	P(H)	P(D|H)	P(H)P(D|H)	P(H|D)
You have Cancer	0.01	0.98	0.0098	
You don't have Cancer	0.99	0.02	0.0198	
			P(D) = .0297	

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Hypothesis	P(H)	P(D|H)	P(H)P(D|H)	P(H|D)
You have Cancer	0.01	0.98	0.0098	0.0098/0.0297 = 0.33
You don't have Cancer	0.99	0.02	0.0198	0.0198/0.0297 = 0.66
			P(D) = .0297	